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On orbifold compactification of $\mathcal{N} = 2$ supergravity in five dimensions

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Abstract: We study compactification of five dimensional ungauged $\mathcal{N} = 2$ supergravity coupled to vector- and hypermultiplets on orbifold $S^1/\mathbb{Z}_2$. In the model, the vector multiplets scalar manifold is arbitrary while the hypermultiplet scalars span a generalized self dual Einstein manifold constructed by Calderbank and Pedersen. The bosonic and the fermionic sectors of the low energy effective $\mathcal{N} = 1$ supergravity in four dimensions are derived.

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Contents

1. Introduction

Compactification of the five dimensional $\mathcal{N} = 2$ supersymmetry on singular space $S^1/\mathbb{Z}_2$ has achieved phenomenological interest since it provides $\mathcal{N} = 1$ supersymmetry in four dimensions. Furthermore, four dimensional $\mathcal{N} = 1$ vacua for the supersymmetric versions of the two branes Randall-Sundrum scenario of the five dimensional $\mathcal{N} = 2$ supergravity has been obtained in [1]. In this scenario one places two 3-branes with opposite tension at the orbifold $S^1/\mathbb{Z}_2$ which is the boundaries of (4+1)-dimensional Anti de Sitter spacetime. The distance between the branes is set by the expectation value of a modulus field, called radion. Furthermore, starting from the simplest model, namely pure supergravity theory one can derive the effective $\mathcal{N} = 1$ theory as it was shown in [2].

A model consists of single hypermultiplet whose moduli space of toric self-dual Einstein (TSDE) in five dimensional $\mathcal{N} = 2$ supergravity has been studied to construct domain wall solutions [3]. They investigated the associated supersymmetric flows to prove the existence of domain walls which admit Randall-Sundrum flows. However, in this model, the Kähler subspace of the TSDE is still unclear.

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The purpose of this paper is to obtain a four dimensional $\mathcal{N}=1$ theory via $S^1/\mathbb{Z}_2$ compactification of the five dimensional $\mathcal{N}=2$ supergravity coupled to arbitrary vector multiplets and a hypermultiplet which is chosen to be a generalized self dual Einstein manifold admitting torus symmetry constructed by Calderbank and Pedersen [4]. Our aim is to find the Kähler subspace of toric self dual Einstein (TSDE) spaces.

Our starting point is the five dimensional $\mathcal{N}=2$ supergravity coupled to arbitrary vector multiplets and a hypermultiplet. First, the theory can be compactified down to four dimensions along an S^1 of radius R parametrized by x_5, resulting in a nonchiral four dimensional $\mathcal{N}=2$ theory. However, we are interested in the chiral four dimensional $\mathcal{N}=1$ theory. Second, to obtain a chiral four dimensional $\mathcal{N}=1$ theory, we consider compactification of the x_5 coordinate on the $S^1/\mathbb{Z}_2$ orbifold. The $\mathbb{Z}_2$ action is as usual $x_5 \to -x_5$ and two fixed points are at $x_5 = 0$ and $x_5 = \pi R$. We begin to mod S^1 by $\mathbb{Z}_2$. In order the reduction to be consistent, we must first make a certain parity assignment to the fields such that the lagrangian is invariant under $x_5 \to -x_5$. Then, when S^1 is modded out by $\mathbb{Z}_2$, only the even parity fields survive on the two fixed points.

The paper is organized as follows. Section 2 briefly reviews the ungauged five dimensional $\mathcal{N}=2$ supergravity coupled to vector- and hypermultiplets. We present the action of the ungauged five dimensional $\mathcal{N}=2$ supergravity. Section 3 presents a detailed derivation of the compactification of five dimensional $\mathcal{N}=2$ supergravity coupled to vector multiplets and hypermultiplets. First, we discuss some basic analysis of the $S^1/\mathbb{Z}_2$ orbifold compactification. The starting point is the bosonic sector of the five dimensional $\mathcal{N}=2$ supergravity theory. Second, after modding out $\mathbb{Z}_2$, the odd fields are projected out and the surviving fields fit into multiplets of the chiral four dimensional $\mathcal{N}=1$ supergravity. The boundary action arising from dimensional reduction can be constructed in a straightforward way and it is obtained by truncating four dimensional $\mathcal{N}=1$ supergravity according to the $\mathbb{Z}_2$ projection. We present then the resulting of the compactification of the bosonic and the fermionic sector. We conclude our results in section 4. Finally, in appendices A, B and C we summarize our notation, convention and some of the detailed calculations.

2. Ungauged five dimensional $\mathcal{N}=2$ supergravity

This section describes supergravity theory in five dimensions with $\mathcal{N}=2$ supersymmetry in which a supergravity multiplet is coupled to matter multiplets. The coupling to vector multiplets was given in [5, 6] and the addition of tensor multiplets was considered in [7]. Furthermore, the full couplings of $\mathcal{N}=2$ supergravity theory in five dimensions was constructed in [8].[1]

2.1 Pure gravitational multiplet

Five dimensional gravitational multiplet consists of the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$, doublet symplectic Majorana gravitinos $\psi^i_{\hat{\mu}}$ and a vector field $\hat{A}_{\hat{\mu}}$ (graviphoton) [10]. The greek hatted indices are five dimensional space-time indices and run over values $0, \ldots, 3, 5$. The index i of the gravitinos runs from 1 to 2.

[1]See also [9] and references therein.

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The bosonic part of the action for the gravitational multiplet takes the form:

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\left[\hat{R} + \hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}} + \frac{1}{6\sqrt{2}\sqrt{-\hat{g}}}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}\hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}_{\hat{\rho}\hat{\sigma}}\hat{A}_{\hat{\lambda}}\right]. \tag{2.1}$$

Furthermore, one can also couple the pure $\mathcal{N} = 2$ gravitational multiplet to arbitrary number of vector- and hypermultiplets. This will be discussed in the next section.

2.2 Couplings of vector- and hypermultiplets

2.2.1 The scalar manifold

First, let us describe n_V vector multiplets of $\mathcal{N} = 2$ supergravity.[2] We now have $n_V + 1$ vector fields A^I_μ, n_V symplectic pairs of gauginos λ^{ia}, and n_V real scalars ϕ^x. It is convenient to group vectors with the graviphoton so that the index $I = 0, 1, \ldots, n_V$ and $a = 1, \ldots, n_V$ are corresponding flat space indices. The kinetic term of the scalars defines the sigma model:

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}g_{xy}(\phi)\partial_{\hat{\mu}}\phi^x\partial^{\hat{\mu}}\phi^y\,. \tag{2.2}$$

The vector multiplet scalars ϕ^x, $x = 1, \ldots, n_V$, parametrize the target space $\mathcal{S}$ where x represents curved indices. The metric g_{xy} can be interpreted as a metric on a riemannian manifold $\mathcal{S}$ called the very special real geometry because it can be viewed as a hypersurface by an n_V polynomial of degree three

$$N(h) = C_{IJK}h^Ih^Jh^K = 1\,, \tag{2.3}$$

where $h^I = h^I(\phi^x)$. Moreover, the gauge coupling of the theory can be expressed as

$$a_{IJ}(h) = -\frac{1}{3}\frac{\partial}{\partial h^I}\frac{\partial}{\partial h^J}\ln N(h)|_{N=1}\,. \tag{2.4}$$

Restricting to the submanifold we can then write the metric $g_{xy}(\phi)$ as:

$$g_{xy}(\phi) = \frac{\partial h^I}{\partial\phi^x}\frac{\partial h^J}{\partial\phi^y}a_{IJ}(h)\,. \tag{2.5}$$

Secondly, we discuss n_H hypermultiplets in which it contains $4n_H$ real scalars q^X and $2n_H$ symplectic Majorana fermions (hyperinos) ζ^A where $X = 1, \ldots, 4n_H$ and $A = 1, \ldots, 2n_H$. As in the previous case, the central object is the metric g_{XY} of the sigma model:

$$\mathcal{L}_{\rm kin} = -\frac{1}{2}g_{XY}(q)\partial_{\hat{\mu}}q^X\partial^{\hat{\mu}}q^Y\,. \tag{2.6}$$

Again, g_{XY} is the metric on a riemannian manifold $\mathcal{Q}$ on which the scalars q^X are the coordinates and thus $X = 1, \ldots, 4n_H$ are the curved indices labelling the coordinates. Local supersymmetry further implies that $\mathcal{Q}$ has to be a quaternionic Kähler manifold [11].

Thus from the above discussion it shows that the scalar manifold $\mathcal{M}$ is a direct product of a very special manifold $\mathcal{S}$ and a quaternionic manifold $\mathcal{Q}$:

$$\mathcal{M} = \mathcal{S}\otimes\mathcal{Q}\,, \tag{2.7}$$

with $\phi^x \in \mathcal{S}$, $q^X \in \mathcal{Q}$.

[2] We omit tensor multiplets for simplicity. For $\mathcal{N} = 2$ supergravity coupled to tensor multiplets see [7].

We now write the action of $\mathcal{N}=2$ supergravity which is needed for our analysis[3]

$$\begin{aligned} S = \int d^5x\sqrt{-\hat{g}}\Bigg[&\frac{1}{2\kappa_5^2}\hat{R} - \frac{1}{4}a_{IJ}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^{J\hat{\mu}\hat{\nu}} - \frac{1}{2}g_{xy}\partial_{\hat{\mu}}\phi^x\partial^{\hat{\mu}}\phi^y - \frac{1}{2}g_{XY}\partial_{\hat{\mu}}q^X\partial^{\hat{\mu}}q^Y - \\ &- \frac{1}{2\kappa_5^2}\bar{\psi}_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi_{\hat{\nu}} - \frac{1}{2}\bar{\lambda}_x\gamma^{\hat{\mu}}D_{\hat{\mu}}\lambda^x - \bar{\zeta}^A\gamma^{\hat{\mu}}D_{\hat{\mu}}\zeta_A + \\ &+ \frac{1}{6\sqrt{6}\sqrt{-\hat{g}}}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}C_{IJK}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^J_{\hat{\rho}\hat{\sigma}}\hat{A}^K_{\hat{\lambda}} - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^{\hat{c}\hat{d}I}\bar{\psi}^{\hat{a}}\gamma_{\hat{a}\hat{b}\hat{c}\hat{d}}\psi^{\hat{b}} + \\ &+ \frac{i\kappa_5}{4}\sqrt{\frac{2}{3}}\left(\frac{1}{4}g_{xy}h_I + T_{xyz}h^z_I\right)\lambda^x\gamma^{\hat{a}\hat{b}}\hat{F}^I_{\hat{a}\hat{b}}\lambda^y + \frac{i\kappa_5}{8}\sqrt{6}h_I\bar{\zeta}_A\gamma^{\hat{a}\hat{b}}\hat{F}^I_{\hat{a}\hat{b}}\zeta^A\Bigg], \end{aligned} \tag{2.8}$$

where the covariant derivatives are given by

$$D_{\hat{\mu}}\lambda^{xi} = \partial_{\hat{\mu}}\lambda^{xi} + \partial_{\hat{\mu}}\phi^y\Gamma^x_{yz}\lambda^{zi} + \frac{1}{4}\omega_{\hat{\mu}}{}^{\hat{a}\hat{b}}\gamma_{\hat{a}\hat{b}}\lambda^{xi} + \partial_{\hat{\mu}}q^X\omega_{Xj}{}^i\lambda^{xj}, \tag{2.9}$$

$$D_{\hat{\mu}}\zeta^A = \partial_{\hat{\mu}}\zeta^A + \partial_{\hat{\mu}}q^X\omega_{XB}{}^A\zeta^B + \frac{1}{4}\omega_{\hat{\mu}}{}^{\hat{b}\hat{c}}\gamma_{\hat{b}\hat{c}}\zeta^A, \tag{2.10}$$

$$D_{\hat{\mu}}\psi^i_{\hat{\nu}} = \left(\partial_{\hat{\mu}} + \frac{1}{4}\omega_{\hat{\mu}}{}^{\hat{b}\hat{c}}\gamma_{\hat{b}\hat{c}}\right)\psi^i_{\hat{\nu}} - \partial_{\hat{\mu}}q^X\omega_X{}^{ij}\psi_{\hat{\nu}j}. \tag{2.11}$$

2.2.2 Toric self dual Einstein spaces

Let us now consider the hypermultiplet sector that is four dimensional ($n_H = 1$) which has self dual property beside Einstein spaces. Our choice is the most general space admitting T^2 isometry which has been shown in [4]. The metric has the form

$$\begin{aligned} ds^2 = &-\left[\frac{1}{4\rho^2} - \frac{(f_\rho^2 + f_\eta^2)}{f^2}\right]\left(d\rho^2 + d\eta^2\right) - \\ &- \frac{\left((f - 2\rho f_\rho)\alpha - 2\rho f_\eta\beta\right)^2 + \left(-2\rho f_\eta\alpha + (f + 2\rho f_\rho)\beta\right)^2}{f^2(f^2 - 4\rho^2(f_\rho^2 + f_\eta^2))}, \end{aligned} \tag{2.12}$$

where $\alpha = \sqrt{\rho}d\phi$ and $\beta = (d\psi + \eta d\phi)$. The function $f(\rho, \eta)$ satisfies the Laplace equation in two dimensional hyperbolic space spanned by (ρ, η)

$$\rho^2(f_{\rho\rho} + f_{\eta\eta}) = \frac{3}{4}f, \tag{2.13}$$

with $f_{\rho\rho} = \partial^2 f/\partial\rho^2$ and $f_{\eta\eta} = \partial^2 f/\partial\eta^2$. One takes $\rho > 0$ and $\eta \in \mathbb{R}$ while (ϕ, ψ) are periodic coordinates. Furthermore, it has positive scalar curvature if f satisfies $f^2 > 4\rho^2(f_\rho^2 + f_\eta^2)$ and negative if $f^2 < 4\rho^2(f_\rho^2 + f_\eta^2)$.

3. Compactification on the orbifold $S^1/\mathbb{Z}_2$

3.1 Analysis of the orbifold transformation

Now we turn our attention to consider the five dimensional supergravity on the orbifold $S^1/\mathbb{Z}_2$.[4] As we shall see that the five dimensional $\mathcal{N}=2$ supergravity is reduced to four dimensional $\mathcal{N}=1$ supergravity.

[3] For complete action see [9].

[4] We discuss S^1 compactification in appendix B.

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There are two ways to employ the compactification on orbifold. In the so-called downstair approach, we consider the five dimensional supergravity with $x^5 \in S^1/\mathbb{Z}_2 = [0, \pi R]$, so $x^5 \sim x^5 + 2\pi R$ and $x^5 \sim -x^5$. Then, the five dimensional space-time is $M^5_{down} = M^4 \times (S^1/\mathbb{Z}_2) = M^4 \times [0, \pi R]$. Locally, we still have ordinary the five dimensional supergravity, but on boundary special thing can occur. The boundary consist of two four dimensional space-time, one at $x^5 = 0$ called M^4 and one at $x^5 = \pi R$ called M'^4.

In the so called upstair approach, we consider the five dimensional supergravity with x^5 on S^1 so that $x^5 \sim x^5 + 2\pi R$ and the five dimensional space-time is $M^5_{up} = M^4 \times S^1$. Then, we define the orbifold transformation $\mathcal{O}$ by

$$\mathcal{O} : x^5 \to -x^5 , \qquad x^\mu \to x^\mu . \tag{3.1}$$

The fixed points of $\mathcal{O}$ are the points with $x^5 = 0$ or $x^5 = \pi R$, so the fixed points consist of two four dimensional space-time M^4 and M'^4 which are the boundary of M^5_{up}. To get the same picture as in the downstair approach, we demand that the physics should be invariant under the orbifold transformation $\mathcal{O}$. We must in particular have the distance $d\hat{s}^2$ invariant,

$$\begin{aligned} d\hat{s}^2 &= \hat{g}_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} \\ &= \hat{g}_{\mu\nu} dx^\mu dx^\nu + 2\hat{g}_{\mu 5} dx^\mu dx^5 + \boldsymbol{\hat{g}_{55} dx^5 dx^5} \\ &\to \hat{g}_{\mu\nu} dx^\mu dx^\nu + 2\hat{g}_{\mu 5} dx^\mu (-dx^5) + \hat{g}_{55}(-dx^5)(-dx^5) . \end{aligned} \tag{3.2}$$

The invariant properties of the distance require that

$$\hat{g}_{\mu\nu} \to \hat{g}_{\mu\nu} , \qquad \hat{g}_{\mu 5} \to -\hat{g}_{\mu 5} , \qquad \hat{g}_{55} \to \hat{g}_{55} , \tag{3.3}$$

such that on M^4 and M'^4, we have that $\hat{g}_{\mu 5} = 0$.

Next, we analyze the gravitational part of the five dimensional supergravity action. Under parity transformation, the action become

$$\int d^5x \sqrt{-\hat{g}} \hat{R} \to \int dx^4 (-dx^5) \sqrt{-\hat{g}} \hat{R} = - \int d^5x \sqrt{-\hat{g}} \hat{R} , \tag{3.4}$$

where R and $\sqrt{-g}$ are invariant under $x^5 \to -x^5$. Therefore, we need that the other terms change sign as well.

The transformation of $\hat{F}^I_{\mu\nu}$ can be seen directly from the transformation of $\hat{g}^{\mu 5}$

$$\begin{aligned} \hat{F}^I_{\mu\nu} &= \partial_\mu \hat{A}^I_\nu - \partial_\nu \hat{A}^I_\mu \to -\hat{F}^I_{\mu\nu} , \\ \hat{F}^I_{\mu 5} &= \partial_\mu \hat{A}^I_5 - \partial_5 \hat{A}^I_\mu \to \hat{F}^I_{\mu 5} , \end{aligned} \tag{3.5}$$

from which it can be checked that FF-term changes sign:

$$\begin{aligned} S_{FF} &= \int d^5x \sqrt{-g} a_{IJ} \hat{F}^I_{\hat{\mu}\hat{\nu}} \hat{F}^{\hat{\mu}\hat{\nu} J} \\ &= \int d^5x \sqrt{-\hat{g}} a_{IJ} \hat{F}^I_{\hat{\mu}\hat{\nu}} \hat{F}^J_{\hat{\rho}\hat{\sigma}} \hat{g}^{\hat{\mu}\hat{\rho}} \hat{g}^{\hat{\nu}\hat{\sigma}} \end{aligned}$$

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$$\rightarrow \int(-d^5x)\sqrt{-\hat{g}}a_{IJ}(-\hat{F}^I_{\mu\nu})(-\hat{F}^J_{\rho\sigma})\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma}+$$
$$+2\int(-d^5x)\sqrt{-\hat{g}}a_{IJ}(\hat{F}^I_{\mu 5})(\hat{F}^J_{\nu 5})(-\hat{g}^{\mu 5})(-\hat{g}^{\nu 5})$$
$$= -\int d^5x\sqrt{-\hat{g}}a_{IJ}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}J}\,. \tag{3.6}$$

Finally, we have $\hat{A}^I_\mu \rightarrow -\hat{A}^I_\mu$ and $\hat{A}^I_5 \rightarrow \hat{A}^I_5$.

Since the orbifold $S^1/\mathbb{Z}_2$ has boundaries at the two fixed points, we have to add the extra terms to the action (2.8) and then get the modified action. First of all, we derive the equation of motion including variation of Ricci tensor. In other words, the derivative of the variation of the metric is not zero on the boundary. In the following we only consider variation of Ricci scalar which give the modified action of the five dimensional supergravity theory on orbifold, and then obtain the modified action of the five dimensional supergravity which can be written as

$$S^{\rm mod}_{5d} = S + 2\int_{\partial\Sigma}\hat{E}\,, \tag{3.7}$$

where $\hat{E} = \hat{h}^{\hat{\mu}}{}_{\hat{\nu}}\nabla_{\hat{\mu}}\hat{N}^{\hat{\nu}}$ is the trace of the extrinsic curvature. In next section we discuss the second term of the above equation.

The variation of the five dimensional Ricci scalar is given by

$$\delta\hat{S} = \int d^5x\Big[\delta\sqrt{-\hat{g}}\hat{R} + \sqrt{-\hat{g}}\delta\hat{g}^{\hat{\mu}\hat{\nu}}\hat{R}_{\hat{\mu}\hat{\nu}} + \sqrt{-\hat{g}}\hat{g}^{\hat{\mu}\hat{\nu}}\delta\hat{R}_{\hat{\mu}\hat{\nu}}\Big]$$
$$= \int d^5x\sqrt{-\hat{g}}\left[g\frac{1}{2}\hat{g}^{\hat{\rho}\hat{\sigma}}\hat{R} - \hat{g}^{\hat{\mu}\hat{\rho}}\hat{g}^{\hat{\sigma}\hat{\nu}}\hat{R}_{\hat{\mu}\hat{\nu}}\right]\delta\hat{g}_{\hat{\rho}\hat{\sigma}} +$$
$$+\int d^5x\sqrt{-\hat{g}}\nabla^{\hat{\rho}}\Big[-\nabla_{\hat{\rho}}(\hat{g}^{\hat{\mu}\hat{\nu}}\delta\hat{g}_{\hat{\mu}\hat{\nu}}) + \nabla^{\hat{\mu}}\delta\hat{g}_{\hat{\rho}\hat{\mu}}\Big]\,, \tag{3.8}$$

where we have used

$$\delta\hat{R}_{\hat{\mu}\hat{\nu}} = \frac{1}{2}\hat{g}^{\hat{\rho}\hat{\sigma}}\Big(-\nabla_{\hat{\mu}}\nabla_{\hat{\nu}}(\delta\hat{g}_{\hat{\rho}\hat{\sigma}}) + \nabla_{\hat{\rho}}\nabla_{\hat{\nu}}(\delta\hat{g}_{\hat{\sigma}\hat{\mu}}) + \nabla_{\hat{\rho}}\nabla_{\hat{\mu}}(\delta\hat{g}_{\hat{\sigma}\hat{\nu}}) - \nabla_{\hat{\rho}}\nabla_{\hat{\sigma}}(\delta\hat{g}_{\hat{\mu}\hat{\nu}})\Big)\,, \tag{3.9}$$

and the metric postulate $\nabla_{\hat{\rho}}g_{\hat{\mu}\hat{\nu}} = 0$. After plugging (3.9) into (3.8), finally we get

$$\delta\hat{S} = \int d^5x\sqrt{-\hat{g}}\left[\frac{1}{2}\hat{g}^{\hat{\rho}\hat{\sigma}}\hat{R} - \hat{R}^{\hat{\rho}\hat{\sigma}}\right]\delta\hat{g}_{\hat{\rho}\hat{\sigma}} + \int d^5x\sqrt{-\hat{g}}\nabla^{\hat{\mu}}\hat{T}_{\hat{\mu}}\,, \tag{3.10}$$

where

$$\hat{T}_{\hat{\mu}} = \hat{g}^{\hat{\nu}\hat{\rho}}(\nabla_{\hat{\rho}}\delta\hat{g}_{\hat{\mu}\hat{\nu}} - \nabla_{\hat{\mu}}\delta\hat{g}_{\hat{\nu}\hat{\rho}})\,. \tag{3.11}$$

Using Gauss theorem, we can rewrite the last term of (3.10):

$$\int_\Sigma d^5x\partial^{\hat{\mu}}(\sqrt{-\hat{g}}\hat{T}_{\hat{\mu}}) = \int_{\partial\Sigma} d^4x\sqrt{-\hat{g}}\hat{N}^{\hat{\mu}}\hat{T}_{\hat{\mu}}\,, \tag{3.12}$$

where $\hat{N}^{\hat{\mu}}$ is the normal to the surface. This term is zero because the derivative of the variation of the metric is zero on the boundary. This term modifies the action of the five dimensional supergravity theory on orbifold.

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Next, after assuming $\delta\hat{g}_{\hat{\mu}\hat{\nu}}$ is a constant on the surface $\partial\Sigma$, then the equation (3.12) can be written down as:

$$\int_{\Sigma} d^5x\partial^{\hat{\mu}}(\sqrt{-\hat{g}}\hat{T}_{\hat{\mu}}) = -\int_{\partial\Sigma} d^4x\sqrt{-\hat{g}}\hat{N}^{\hat{\mu}}\hat{h}^{\hat{\nu}\hat{\rho}}\nabla_{\hat{\mu}}(\delta\hat{g}_{\hat{\nu}\hat{\rho}})\,, \tag{3.13}$$

where we have used $\hat{h}^{\hat{\nu}\hat{\rho}}\nabla_{\hat{\rho}}\delta\hat{g}_{\hat{\mu}\hat{\nu}} = 0$. The boundaries can be considered as a four dimensional surface embedded in the five dimensional space-time.

We now define a quantity $\hat{E}$, whose variation equals to the equation (3.13). We have

$$\hat{E}_{\hat{\mu}\hat{\nu}} = \hat{h}_{\hat{\mu}}{}^{\hat{\rho}}\nabla_{\hat{\rho}}\hat{N}_{\hat{\nu}}\,, \qquad \hat{h}_{\hat{\mu}\hat{\nu}} = \hat{g}_{\hat{\mu}\hat{\nu}} - \hat{N}_{\hat{\mu}}\hat{N}_{\hat{\nu}}\,, \tag{3.14}$$

where

$$\hat{N}_{\hat{\mu}} = \pm\delta^5_{\hat{\mu}}\sqrt{\hat{g}_{55}}\,. \tag{3.15}$$

From the above equation we see that induced metric on the surface $\partial\Sigma$ has

$$\hat{h}_{55} = 0\,. \tag{3.16}$$

The trace of extrinsic curvature $\hat{E}$ is calculated according to

$$\begin{aligned}\hat{E} &= \hat{g}^{\hat{\mu}\hat{\nu}}\hat{E}_{\hat{\mu}\hat{\nu}} = \hat{g}^{\hat{\mu}\hat{\nu}}\hat{h}_{\hat{\mu}}{}^{\hat{\rho}}\nabla_{\hat{\rho}}\hat{N}_{\hat{\nu}}\\ &= \hat{h}^{\hat{\nu}\hat{\rho}}(\partial_{\hat{\rho}}\hat{N}_{\hat{\nu}} - \hat{\Gamma}^{\hat{\sigma}}_{\hat{\rho}\hat{\nu}}\hat{N}_{\hat{\sigma}}) = \hat{h}^{5\hat{\rho}}(\partial_{\hat{\rho}}\hat{N}_5 - \hat{\Gamma}^5_{\hat{\rho}\hat{\nu}}\hat{N}_5)\\ &= \hat{h}^{5\hat{\rho}}\partial_{\hat{\rho}}\hat{N}_5 - \frac{1}{2}\hat{g}^{5\hat{\lambda}}\hat{h}^{\hat{\nu}\hat{\rho}}[\partial_{\hat{\rho}}\hat{g}_{\hat{\lambda}\hat{\nu}} - \partial_{\hat{\nu}}\hat{g}_{\hat{\lambda}\hat{\rho}} + \partial_{\hat{\lambda}}\hat{g}_{\hat{\nu}\hat{\rho}}]\hat{N}_5\,,\end{aligned} \tag{3.17}$$

where we have used definition of the Christoffel symbol.

Now we use the fact that both $\hat{g}_{\hat{\mu}\hat{\nu}}$ and $\hat{h}_{\hat{\mu}\hat{\nu}}$ are block diagonal

$$\begin{aligned}\hat{E} &= \hat{h}^{55}\partial_5\hat{N}_5 - \frac{1}{2}\hat{g}^{55}\hat{h}^{5\hat{\rho}}\partial_{\hat{\rho}}\hat{g}_{55} - \frac{1}{2}\hat{g}^{55}\hat{h}^{\hat{\nu}5}\partial_{\hat{\nu}}\hat{g}_{55} + \frac{1}{2}\hat{g}^{55}\hat{h}^{\hat{\nu}r\hat{h}o}\partial_5\hat{g}_{\hat{\nu}\hat{\rho}}\hat{N}_5\\ &= \frac{1}{2}\hat{g}^{55}\hat{h}^{\hat{\nu}\hat{\rho}}\partial_5\hat{g}_{\hat{\nu}\hat{\rho}}\hat{N}_5,\end{aligned} \tag{3.18}$$

where we have used the fact that $\hat{h}^{\hat{\nu}5}\partial_{\hat{\nu}}\hat{g}_{55} = 0$ since this is a derivative along the surface and that the variation is zero on the surface. The second term of (3.7) comes from variation of the extrinsic curvature (3.17), we get

$$\delta\hat{E} = \frac{1}{2}\hat{N}^{\hat{\mu}}\hat{h}^{\hat{\nu}\hat{\rho}}(\nabla_{\hat{\mu}}\delta\hat{g}_{\hat{\nu}\hat{\rho}})\,, \tag{3.19}$$

which is identical to (3.13) apart from the factor two. Here we are working in the massless sector of the theory, where $\partial_5\hat{g}_{\hat{\mu}\hat{\nu}} = 0$ so the second term of (3.7) does not contribute to our action. However if we are working in the massive sector, this term can not be ignored. In the following section we only consider the massless sector to obtain the low-energy effective action of the four dimensional $\mathcal{N} = 1$ supersymmetry theory.

From the bosonic sector analysis, the five dimensional fields can be even ($\Phi(x^5) = \Phi(-x^5)$) or odd ($\Phi(x^5) = -\Phi(-x^5)$) under orbifold transformation. Note that the odd fields must either vanish or be discontinuous at the fixed points, hence they are not dynamical fields on the submanifolds. Identifying periodicity of the scalar fields in (2.12),

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$\Phi \to \Phi + \Phi_0$ with $\Phi \equiv (\rho, \eta, \psi, \phi)$ and under orbifold transformation $\Phi \to -\Phi$, we define the fields (ρ, η) are even and (ψ, ϕ) are odd.[5]

The action of Z_2 on the fermion fields and on the spinor parameter ϵ of the supersymmetry transformations is defined as [12, 13]:

$$\psi^i_\mu(x^5) = \mathbf{P}(\psi)\gamma_5 M(q)^i{}_j \psi^j_\mu(-x^5)\,, \tag{3.20}$$

$$\lambda^i(x^5) = \mathbf{P}(\lambda)\gamma_5 M(q)^i{}_j \lambda^j(-x^5)\,, \tag{3.21}$$

$$\epsilon^i(x^5) = \mathbf{P}(\epsilon)\gamma_5 M(q)^i{}_j \varepsilon^j(-x^5)\,, \tag{3.22}$$

where

$$M^i{}_j = m_1(q)(\sigma_1)^i{}_j + m_2(q)(\sigma_2)^i{}_j + m_3(q)(\sigma_3)^i{}_j\,, \tag{3.23}$$

with $m_1, m_2, m_3 \in$ real functions of q and $\mathbf{P}(\Psi) = \pm$, $\Psi \equiv (\lambda, \psi, \varepsilon)$. Decomposing the five dimensional spinor Ψ, and its conjugate $\bar{\Psi}$, into four-dimensional spinor, and following the convention in [2] it is given by

$$\Psi_{\hat{\mu}} = \begin{pmatrix} \psi^1_{\hat{\mu}L} \\ \overline{\psi}_{2\hat{\mu}R} \end{pmatrix}, \tag{3.24}$$

and

$$\gamma^5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \qquad \gamma^{\hat{a}} = \begin{pmatrix} 0 & \sigma^{\hat{a}} \\ \sigma^{\hat{a}} & 0 \end{pmatrix}. \tag{3.25}$$

To complete the results of this section, we must first take certain parity assignments to the fields such that the action stays invariant under $x^5 \to -x^5$. Then, when we mod out $\mathbb{Z}_2$, only fields of even parity survive on the two orbifold fixed points. The even parity fields are given by

$$\hat{g}_{\mu\nu}\,, \quad \hat{g}_{55}\,, \quad \hat{A}^I_5\,, \quad \rho\,, \quad \eta\,, \quad \psi^+_\mu\,, \quad \psi^-_5\,, \quad \zeta^1\,, \quad \lambda^1_x\,, \quad \epsilon^+\,, \tag{3.26}$$

and those of odd parity are given by

$$\hat{g}_{\mu 5}\,, \quad \hat{g}_{5\mu}\,, \hat{A}^I_\mu\,, \quad \phi\,, \quad \psi\,, \quad \psi^-_\mu\,, \quad \psi^+_5\,, \quad \zeta^2\,, \quad \lambda^2_x\,, \quad \epsilon^-\,, \tag{3.27}$$

where we define $\psi^{\pm}_{\hat{\mu}} = \frac{1}{\sqrt{2}}(\psi^1_{\hat{\mu}} \pm \psi^2_{\hat{\mu}})$.

3.2 The bosonic sector

In this subsection we derive the low energy effective $\mathcal{N} = 1$ action via compactification of the bosonic part of the action of the ungauged $\mathcal{N} = 2$ supergravity in five dimensions (2.8) on the orbifold $S^1/\mathbb{Z}_2$ using the analysis above.

Under the $\mathbb{Z}_2$ symmetry, the bosonic fields $(\hat{g}_{\mu\nu}, \hat{g}_{55}, \hat{A}^I_5, \rho, \eta)$ have to be even, while $(\hat{g}_{\mu 5}, \hat{A}^I_\mu, \phi, \psi)$ are odd with respect to the orbifold transformation. The analysis of the

[5] The odd parity of (ρ, η) is not satisfied in the function $F(\rho, \eta)$, for example, for $F(\rho, \eta) = \frac{\sqrt{\rho^2 + (\eta - y)^2}}{\sqrt{\rho}}$. See [4].

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orbifold transformation above are similar to S^1 compactification,[6] however all odd fields are ruled out. Using the ansatz,

$$d\hat{s}^2 = A(x) g_{\mu\nu} dx^\mu dx^\nu + B(x) dz^2 \,, \tag{3.28}$$

we find

$$\begin{aligned} S_{S^1/\mathbb{Z}_2} = \int d^4\sqrt{-g}\Bigg[& \frac{1}{2\kappa_4^2}\left(AB^{1/2}R + \frac{3}{2}A^{-1}B^{1/2}\partial_\mu A\partial^\mu A + \frac{3}{2}B^{1/2}\partial_\mu A\partial^\mu B\right) - \\ & - \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right) AB^{-1/2} a_{IJ}\partial_\mu A_5^I \partial^\mu A_5^I - \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right) AB^{1/2} g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y + \\ & + \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right) AB^{1/2}\left(\frac{1}{4\rho^2} - \frac{f_\rho^2+f_\eta^2}{f^2}\right)\Big(\partial_\mu\rho\partial^\mu\rho + \partial_\mu\eta\partial^\mu\eta\Big)\Bigg] . \end{aligned} \tag{3.29}$$

To put the equation (3.29) back into the canonical form, we perform a Weyl rescaling of the metric which is given by:

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = A^{-1}B^{-1/2} g_{\mu\nu} \,, \qquad g^{\mu\nu} \to \tilde{g}^{\mu\nu} = AB^{1/2} g^{\mu\nu} \,. \tag{3.30}$$

So under this rescaling the equation (3.29) becomes:

$$\begin{aligned} S_{S^1/\mathbb{Z}_2} = \int d^4\sqrt{-g}\Bigg[& \frac{1}{2\kappa_4^2}\left(R - \frac{3}{8}\partial_\mu \ln B\partial^\mu \ln B\right) - \\ & - \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right) B^{-1} a_{IJ}\partial_\mu A_5^I \partial^\mu A_5^I - \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right) g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y + \\ & + \frac{1}{2}\left(\frac{\kappa_5^2}{\kappa_4^2}\right)\left(\frac{1}{4\rho^2} - \frac{f_\rho^2+f_\eta^2}{f^2}\right)\Big(\partial_\mu\rho\partial^\mu\rho + \partial_\mu\eta\partial^\mu\eta\Big)\Bigg] . \end{aligned} \tag{3.31}$$

The four dimensional gravitational constant ($1/\kappa_4^2$) can be expressed in terms of its five dimensional counterpart as:

$$\kappa_4^2 = \frac{\kappa_5^2}{2\pi R} \,. \tag{3.32}$$

The equation (3.31) can be rewritten in the form:

$$\begin{aligned} S_{S^1/\mathbb{Z}_2} = \frac{1}{2\kappa_4^2}\int d^4\sqrt{-g}\Bigg[& R - \frac{1}{\kappa_5^2}\hat{a}_{IJ}\partial_\mu\hat{h}^I\partial^\mu\hat{h}^J - \frac{2}{3}\hat{a}_{IJ}\partial_\mu A_5^I\partial^\mu A_5^I + \\ & + \kappa_5^2\left(\frac{1}{4\rho^2} - \frac{f_\rho^2+f_\eta^2}{f^2}\right)\Big(\partial_\mu\rho\partial^\mu\rho + \partial_\mu\eta\partial^\mu\eta\Big)\Bigg] , \end{aligned} \tag{3.33}$$

where $\hat{a}_{IJ} = \frac{3\kappa_5^2}{2}B^{-1}a_{IJ}$, $\hat{h}^I = B^{1/2}h^I$ and we have used the identities

$$h_x^I = -\sqrt{\frac{3}{2\kappa_5^2}}h_{,x}^I \,, \quad h^I h_I^x = 0 \,, \quad g_{xy} = a_{IJ}h_x^I h_y^J \,, \quad a_{IJ} = -2C_{IJK} + 3h_I h_J \,. \tag{3.34}$$

[6] See appendix B.

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In (3.33), n_V+1 vector multiplet scalars ϕ^x appear through n_V+1 scalars $\hat{h}^I$ subject to the constraint,

$$C_{IJK}(\hat{h})\hat{h}^I\hat{h}^J\hat{h}^K = B^{-3/2}\,. \tag{3.35}$$

Now we consider the low-energy effective action of $\mathcal{N}=1$ supergravity in four dimensions. Therefore, we must show that the scalars in (3.33) parametrize a complex manifold of the Kähler type. For that purpose, we define two new complex quantities, T and S:

$$T^I = \frac{1}{\kappa_5}\hat{h}^I + i\sqrt{\frac{3}{2}}A_5^I\,, \tag{3.36}$$

$$S = \frac{1}{\kappa_5}(\rho+i\eta)\,. \tag{3.37}$$

From this definition, the function $f(\rho,\eta)$ is replaced by $f(S,\bar{S})$ and we get

$$\hat{a}_{IJ} = \frac{6}{(T^I+\bar{T}^I)(T^J+\bar{T}^J)}\,, \tag{3.38}$$

$$f_\rho = \frac{1}{\kappa_5}(f_S+f_{\bar{S}})\,, \tag{3.39}$$

$$f_\eta = \frac{i}{\kappa_5}(f_S-f_{\bar{S}})\,. \tag{3.40}$$

The Laplace equation (2.13) can be casted into

$$f_{S\bar{S}} = \frac{3f}{4(S+\bar{S})^2}\,. \tag{3.41}$$

By substituting (3.36) and (3.38) into (3.33), the action can be rewritten as

$$\begin{aligned} S_{S^1/\mathbb{Z}_2} &= \int d^4x\sqrt{-g}\frac{1}{2\kappa_4^2}R - \int d^4x\sqrt{-g}\frac{3}{\kappa_4^2(T^I+T^{\bar{I}})(T^J+T^{\bar{J}})}\partial_\mu T^I\partial^\mu T^{\bar{J}} - \\ &\quad - \int d^4x\sqrt{-g}\frac{1}{\kappa_4^2}\left[\frac{1}{(S+\bar{S})^2}+2\left(\frac{f_Sf_{\bar{S}}}{f^2}-\frac{f_{S\bar{S}}}{f}\right)\right]\partial_\mu S\partial^\mu\bar{S} \\ &= \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}K_{I\bar{J}}\partial_\mu T^I\partial^\mu T^{\bar{J}} - - \int d^4x\sqrt{-g}K_{S\bar{S}}\partial_\mu S\partial^\mu\bar{S}\,, \end{aligned} \tag{3.42}$$

where

$$K_{I\bar{J}} \equiv \frac{\partial}{\partial T^I}\frac{\partial}{\partial T^{\bar{J}}}K_V\,, \tag{3.43}$$

$$K_{S\bar{S}} \equiv \frac{\partial}{\partial S}\frac{\partial}{\partial\bar{S}}K_H\,, \tag{3.44}$$

The Kähler potentials are denoted by K_V and K_H which comes from the vector- and hypermultiplets, respectively. We find that

$$\begin{aligned} K &= K_V + K_H \\ &= -\kappa_4^{-2}\ln(C_{IJK}(T^I+T^{\bar{I}})(T^J+T^{\bar{J}})(T^K+T^{\bar{K}})) - \kappa_4^{-2}(\ln(S+\bar{S})+2\ln f)\,. \end{aligned} \tag{3.45}$$

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3.3 The fermionic sector

In the previous subsection we found the effective action for the bosonic sectors with the Kähler potentials is given by (3.45). The rest is to derive the fermionic sectors of the effective four dimensional $\mathcal{N}=1$ theory. Since the ansatz metric is non-radion background, the fermionic fields do not depend on x^5 and then the integral over the compact dimensions in the action yields just the volume which can be absorbed into the definition of the four dimensional gravitational constant. In other words, it is equivalent to integrate out the massive Kaluza-Klein modes and one keeps only zero modes in the effective description.

Our starting point is the fermionic parts of the action (2.8). The kinetic term of the $\psi\psi$-component is given by

$$S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[-\frac{1}{2\kappa_5^2}\bar{\psi}_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi_{\hat{\nu}} - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\bar{\psi}_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi_{\hat{\nu}}\Big]. \tag{3.46}$$

It is convenient to combine two symplectic Majorana spinors into one even(odd) Majorana spinor in four dimensions, with the following convention

$$\psi_{\hat{\mu}} = \begin{pmatrix}\psi^1_{\hat{\mu}L}\\ \\ \overline{\psi}^2_{\hat{\mu}R}\end{pmatrix}, \qquad \overline{\psi}_{\hat{\mu}} = \left(\psi^2_{\hat{\mu}L}, \overline{\psi}^1_{\hat{\mu}R}\right). \tag{3.47}$$

Using these definitions, we can rewrite (3.46) in terms of even and odd combinations,

$$S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[-\frac{1}{2\kappa_5^2}\psi^2_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^1_{\hat{\nu}} - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\psi^2_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^1_{\hat{\nu}}\Big] + \text{h.c.}\,, \tag{3.48}$$

or

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[&-\frac{1}{2\kappa_5^2}\cdot\frac{1}{2}\cdot\left(\psi^+_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^+_{\hat{\nu}} - \psi^-_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^-_{\hat{\nu}}\right) - \\ &- \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}\left(\psi^+_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^+_{\hat{\nu}} - \psi^-_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^-_{\hat{\nu}}\right)\Big] + \text{h.c.}\,, \end{aligned} \tag{3.49}$$

where we have used

$$\psi^1_{\hat{\mu}} = \frac{1}{\sqrt{2}}\left(\psi^+_{\hat{\mu}} + \psi^-_{\hat{\mu}}\right) \qquad \psi^2_{\hat{\mu}} = \frac{1}{\sqrt{2}}\left(\psi^+_{\hat{\mu}} - \psi^-_{\hat{\mu}}\right). \tag{3.50}$$

The two fixed points require that certain $\mathbb{Z}_2$-odd fields have a step-function, namely $sgn(z = x^5)$. The step function $sgn(z = x^5)$ takes values -1 for $x^5 \in [-\pi R, 0]$ and $+1$ for $x^5 \in [0, \pi R]$.

In order for the reduction to be consistent, we must make an ansatz for the gravitino as follows

$$\psi^+_\mu = \frac{1}{\sqrt{2}}C(x)\psi_\mu\,, \qquad \psi^-_\mu = \frac{1}{\sqrt{2}}sgn(z)C(x)\psi_\mu\,, \tag{3.51}$$

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where $C(x)$ is an arbitrary function. After subtracting to each component and plugging the ansatz into (3.49) we get[7]

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[&-\frac{1}{2\kappa_5^2}\frac{1}{2}\left(\psi_\rho^+\gamma^{\rho\mu\nu}D_\mu\psi_\nu^+ + \psi_\rho^+\gamma^{\rho 5\nu}D_5\psi_\nu^-\right) - \\ &-\frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}\psi_\mu^+\gamma^{\mu\nu\hat{\rho}\hat{\sigma}}\psi_\nu^+ + \\ &+\frac{1}{2\kappa_5^2}\psi_\rho^+\gamma^{\rho\nu}\psi_\nu^+\left(\delta(z)-\delta(z-\pi R)\right)\Big] + \text{h.c.} \end{aligned} \tag{3.52}$$

We note that the third term of the action (3.52) is the boundary term. This result has to be consistent with the upstair approach used in [14] where one regards space-time as the smooth manifold $M_4 \times S^1$ subject to $\mathbb{Z}_2$ symmetry. Then, in the framework of the five dimensional supergravity we can write the total action by

$$S = S_{\text{bulk}} + S_{\text{boundary}}\,, \tag{3.53}$$

where S_{bulk} is given by (2.8) and S_{boundary} or S_{brane} in the context of braneworld is given by

$$S_{\text{brane}} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\psi_\mu^+\gamma^{\mu\nu}\psi_\nu^+[\delta(z)-\delta(z-\pi R)] + \text{h.c.} \tag{3.54}$$

The bulk part of the action (3.52) can be rewritten as follows

$$S_{\psi\psi} = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\psi_\rho\gamma^{\rho\mu\nu}\tilde{D}_\mu\psi_\nu + \text{h.c.}\,, \tag{3.55}$$

where

$$\begin{aligned} \tilde{D}_\mu\psi_\nu = &\ \partial_\mu\psi_\nu + \frac{1}{2}\omega_\mu{}^{ab}\gamma_{ab}\psi_\nu + \partial_\mu \ln C\psi_\nu - \frac{1}{4}\partial_\mu\ln B\psi_\nu - \\ &-\frac{\kappa_4^2}{12}(K_I\partial_\mu T^I - K_{\bar{I}}\partial_\mu T^{\bar{I}})\psi_\nu + \frac{i\kappa_4^2}{2}(K_S\partial_\mu S - K_{\bar{S}}\partial_\mu\bar{S})\psi_\nu\,. \end{aligned} \tag{3.56}$$

After integrating (3.55) with respect to $x^5 \in [-\pi R, \pi R]$, we finally get

$$S_{\psi\psi} = -\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\bar{\psi}_\rho\gamma^{\rho\mu\nu}\tilde{D}_\mu\psi_\nu\,. \tag{3.57}$$

In the above expressions we have set $C = B^{1/4}$ so that the covariant derivative in four dimensions becomes

$$\tilde{D}_\mu\psi_\nu = \partial_\mu\psi_\nu + \frac{1}{2}\omega_\mu{}^{ab}\gamma_{ab}\psi_\nu - \frac{\kappa_4^2}{12}(K_I\partial_\mu T^I - K_{\bar{I}}\partial_\mu T^{\bar{I}})\psi_\nu + \frac{i\kappa_4^2}{2}(K_S\partial_\mu S - K_{\bar{S}}\partial_\mu\bar{S})\psi_\nu\,, \tag{3.58}$$

and then used the spin connections in the metric background,

$$\hat{\omega}_\mu{}^{ab} = \omega_\mu{}^{ab} - \frac{1}{4}(e_\mu^a e^{\nu b} - e_\mu^b e^{\nu a})\partial_\nu \ln B\,, \tag{3.59}$$

$$\hat{\omega}_{5a\bar{5}} = -\frac{1}{2}B^{1/2}e_a^\mu\partial_\mu\ln B\,, \tag{3.60}$$

[7]The detailed calculation is presented in appendix C.

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and Sp(1)-connections,

$$\omega^1 = -\frac{f_\eta}{f} d\rho + \Big(\frac{1}{2\rho} + \frac{f_\rho}{f}\Big) d\eta \,. \tag{3.61}$$

The hyperino kinetic terms of the fermionic sector of the action (2.8) is given by

$$S_{\zeta\zeta} = \int d^5x \sqrt{-\hat{g}} \Big[-\bar{\zeta}^A \gamma^{\hat{\mu}} D_{\hat{\mu}} \zeta_A + \frac{i\kappa_5}{8} \sqrt{6} h_I \bar{\zeta}_A \gamma^{\hat{a}\hat{b}} \hat{F}^I_{\hat{a}\hat{b}} \zeta^A \Big] \,, \qquad A = 1, 2 \,. \tag{3.62}$$

We assume that the fields are independent of the fifth coordinate so that derivative with respect to fifth coordinate vanishes. The ansatz for the hyperino (even fields),

$$\zeta_A \to \zeta_1 = \frac{1}{\sqrt{2}\kappa_5} B^{1/4} \zeta \,, \tag{3.63}$$

and we obtain

$$S_{\zeta\zeta} = -\frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \bar{\zeta} \gamma^\mu \tilde{D}_\mu \zeta \,. \tag{3.64}$$

The covariant derivative for the hyperino in four dimensions is given by

$$\tilde{D}_\mu \zeta = \partial_\mu \zeta + \frac{1}{2} \omega_\mu{}^{ab} \gamma_{ab} \zeta - \frac{i\kappa_4^2}{2} (K_S \partial_\mu S - K_{\bar{S}} \partial_\mu \bar{S}) \zeta + \frac{\kappa_4^2}{6} (K_I \partial_\mu T^I - K_{\bar{I}} \partial_\mu T^{\bar{I}}) \zeta \,. \tag{3.65}$$

Next we look at the gaugino kinetic terms

$$S_{\lambda\lambda} = \int d^5x \sqrt{-\hat{g}} \Bigg[-\frac{1}{2} \bar{\lambda}_x \gamma^{\hat{\mu}} D_{\hat{\mu}} \lambda^x + \frac{i\kappa_5}{4} \sqrt{\frac{2}{3}} \Big(\frac{1}{4} g_{xy} h_I + T_{xyz} h^z_I \Big) \lambda^x \gamma^{\hat{a}\hat{b}} \hat{F}^I_{\hat{a}\hat{b}} \lambda^y \Bigg] \,. \tag{3.66}$$

For gaugino, the procedure is similar to the hyperino terms and we then obtain

$$S_{\lambda\lambda} = -\frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \bar{\lambda}^x \gamma^\mu \tilde{D}_\mu \lambda_x \,, \tag{3.67}$$

where

$$\begin{aligned} \tilde{D}_\mu \lambda_x \, &= \partial_\mu \lambda_x + \frac{1}{2} \omega_\mu{}^{ab} \gamma_{ab} \lambda_x + \partial_\mu \phi^y \Gamma^z_{yx} \lambda_z - \\ &- \frac{i\kappa_4^2}{2} (K_S \partial_\mu S - K_{\bar{S}} \partial_\mu \bar{S}) \lambda_x + \frac{\kappa_4^2}{12} (K_I \partial_\mu T^I - K_{\bar{I}} \partial_\mu T^{\bar{I}}) \lambda_x \,. \end{aligned} \tag{3.68}$$

Finally, by combining the results of the bosonic and the fermionic sectors we get the low-energy effective action of $\mathcal{N} = 1$ in four dimensions:

$$\begin{aligned} S^{N=1}_{d=4} \, &= \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \Big[R + \overline{\psi}_\rho \gamma^{\rho\mu\nu} \tilde{D}_\mu \psi_\nu - \bar{\zeta} \gamma^\mu \tilde{D}_\mu \zeta - \bar{\lambda}^x \gamma^\mu \tilde{D}_\mu \lambda_x - \\ &\qquad - 2\kappa_4^2 K_{I\bar{I}} \partial_\mu T^I \partial^\mu T^{\bar{I}} - 2\kappa_4^2 K_{S\bar{S}} \partial_\mu S \partial^\mu \bar{S} \Big] \,, \end{aligned} \tag{3.69}$$

where the covariant derivative of the spinors are given by (3.58), (3.65) and (3.68).

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4. Conclusions and outlook

In this paper we have studied $S^1/\mathbb{Z}_2$ compactification of the ungauged $\mathcal{N}=1$ supergravity in five dimensions coupled to arbitrary vector multiplets and a hypermultiplet where the scalar fields span toric self dual Eintein spaces. The resulting theory is four dimensional $\mathcal{N}=1$ supergravity. In the bosonic sector we found that Kähler potential from the vector multiplets contribution has the form

$$K_V = -\kappa_4^{-2} \ln\Big(C_{IJK}(T^I + T^{\bar{I}})(T^J + T^{\bar{J}})(T^K + T^{\bar{K}})\Big)\,, \tag{4.1}$$

and from hypermultiplets contribution are

$$K_H = -\kappa_4^{-2}\left(\ln(S+\bar{S}) + 2\ln f\right). \tag{4.2}$$

Furthermore, we have derived the fermionic sectors of the four dimensional $\mathcal{N}=1$ theory. In general, our results confirm those in reference [2], where the effective theory was obtained by compactification from five dimensional supergravity down to four dimensions in the context of Randall-Sundrum scenario.

It is also interesting to extend our scenario to the general case where the $\mathcal{N}=2$ scalar potential reduced to the $\mathcal{N}=1$ scalar potential in terms of holomorphic superpotential. Another problem is to cancel anomaly of the orbifold model and find the gauge group on two orbifold fixed points such that the gauge and the scalar fields residing on the boundaries can be supersymmetrized. This can be done by modifying the boundary action but also one has to modify the supersymmetry transformation laws in both boundary and bulk fields. This problem will be addressed elsewhere.

Note added: during preparation of this manuscript we became aware of the independent work [15] which has some overlaps with our results.

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A. Conventions and notations

The purpose of this appendix is to collect our conventions in this paper. The spacetime metric is taken to have the signature $(-,+,+,+,+)$ while the Ricci scalar is defined to be $R = g^{\mu\nu}\Big[\frac{1}{2}\partial_\rho\Gamma^\rho_{\mu\nu} - \partial_\nu\Gamma^\rho_{\mu\rho} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho}\Big] + \frac{1}{2}\partial_\rho\Big[g^{\mu\nu}\Gamma^\rho_{\mu\nu}\Big]$. The Christoffel symbol is given by $\Gamma^\mu_{\nu\rho} = \frac{1}{2}g^{\mu\sigma}(\partial_\nu g_{\rho\sigma} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho})$ where $g_{\mu\nu}$ is the spacetime metric.

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Indices:

$\hat{\mu}, \hat{\nu} = 0, \ldots, 3, 5$	label curved five dimensional spacetime indices
$\hat{a}, \hat{b} = 0, \ldots, 3, 5$	label flat five dimensional spacetime indices
$\mu, \nu = 0, \ldots, 3$	label curved four dimensional spacetime indices
$a, b = 0, \ldots, 3$	label flat four dimensional spacetime indices
$i, j = 1, 2$	label the fundamental representation of the R-symmetry group $\mathrm{SU}(2) \otimes \mathrm{U}(1)$
$x, y, z = 1, \ldots, n_V$	label the real scalars of the $\mathcal{N} = 2$ vector multiplet
$I, J, K = 0, 1, \ldots, n_V$	label the vector multiplets
$X, Y, Z = 1, \ldots, 4n_H$	label the real scalars of the $\mathcal{N} = 2$ hypermultiplet
$A, B = 1, \ldots, 2n_H$	label the fundamental representation of $\mathrm{Sp}(2n_H)$.

B. Five dimensional supergravity on S^1

In this section, we derive the dimensional reduction of the bosonic part of the five dimensional supergravity action on S^1.[8] The class of four dimensional theories obtained in this way are only a subclass of the general four dimensional $\mathcal{N} = 2$ theories [17]. In general, we can choose the metric to be

$$ds_5^2 = A(x) g_{\mu\nu} dx^\mu dx^\nu + B(x) dz^2 = \hat{g}_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} \tag{B.1}$$

where A and B are arbitrary functions. We have

$$\hat{g}_{\mu\nu} = A(x) g_{\mu\nu} \qquad \hat{g}_{zz} = B(x)\,. \tag{B.2}$$

The gravity term in the supergravity lagrangian reduced to

$$\sqrt{-\hat{\mathbf{g}}}\hat{\mathbf{R}} \sim \sqrt{-\mathbf{g}}\Big[\mathbf{A}\mathbf{B}^{\mathbf{1/2}}\mathbf{R} + \frac{\mathbf{3}}{\mathbf{2}}\mathbf{A}^{-\mathbf{1}}\mathbf{B}^{\mathbf{1/2}}\partial_\mu \mathbf{A} \partial^\mu \mathbf{A} + \frac{\mathbf{3}}{\mathbf{2}}\mathbf{B}^{-\mathbf{1/2}}\partial_\mu \mathbf{A}\partial^\mu \mathbf{B}\Big], \tag{B.3}$$

where $\sim$ means equal up to a total derivative. The FF-term we are going to reduce is

$$\mathcal{L}_{FF} = -\frac{1}{4}\sqrt{-\hat{g}}\, a_{IJ} \hat{F}^I_{\hat{\mu}\hat{\nu}} \hat{F}^{\hat{\mu}\hat{\nu}J}\,. \tag{B.4}$$

[8]This has also been discussed in the appendix of [16].

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In order to get the massless sector we divide $\hat{F}^I_{\hat{\mu}\hat{\nu}}$ into $(\hat{F}^I_{\mu\nu}, \hat{F}^I_{\mu 5})$. Then, we take Fourier expansion of this term and keep only the lowest order terms which should be independent of x^5. We then have that

$$-\frac{1}{4}\sqrt{-\hat{g}}a_{IJ}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}J} = -\frac{1}{4}\sqrt{-g}B^{1/2}a_{IJ}F^I_{\mu\nu}F^{\mu\nu J} - \frac{1}{2}\sqrt{-g}AB^{-1/2}a_{IJ}\partial_\mu A^I_5\partial^\mu A^J_5\,, \quad \text{(B.5)}$$

where A^I_5 are the fifth component of the vector fields. The vector multiplet scalars sector reduce to

$$\mathcal{L}_\phi = -\frac{1}{2}\sqrt{-\hat{g}}g_{xy}\partial_{\hat{\mu}}\phi^x\partial^{\hat{\mu}}\phi^y = -\frac{1}{2}\sqrt{-g}AB^{1/2}g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y\,. \quad \text{(B.6)}$$

The hypermultiplet sector reduce to

$$\mathcal{L}_q = -\frac{1}{2}\sqrt{-\hat{g}}g_{XY}\partial_{\hat{\mu}}q^X\partial^{\hat{\mu}}q^Y = -\frac{1}{2}\sqrt{-g}AB^{1/2}g_{XY}\partial_\mu q^X\partial^\mu q^Y\,. \quad \text{(B.7)}$$

Moreover The Chern-Simon term can be simplified

$$\begin{aligned}\mathcal{L}_{FFA} &= \frac{1}{6\sqrt{6}}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}C_{IJK}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^J_{\hat{\rho}\hat{\sigma}}A^K_{\hat{\lambda}} \\ &= \frac{1}{2\sqrt{6}}\epsilon^{\mu\nu\rho\sigma}C_{IJK}\hat{F}^I_{\mu\nu}\hat{F}^J_{\rho\sigma}A^K_5 \\ &= \frac{1}{2\sqrt{6}}\epsilon^{\mu\nu\rho\sigma}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_5\,. \end{aligned} \quad \text{(B.8)}$$

Collecting the above results, the bosonic part of the reduced supergravity lagrangian (2.8) is

$$\begin{aligned}\frac{1}{\sqrt{-g}}\mathcal{L}^{S^1}_{4d} &= -\frac{1}{2}AB^{1/2}R - \frac{3}{4}\partial_\mu\ln A\partial^\mu\ln A - \frac{3}{4}\partial_\mu\ln A\partial^\mu\ln B - \\ &\quad -\frac{1}{2}AB^{1/2}g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y - \frac{1}{4}B^{1/2}a_{IJ}F^I_{\mu\nu}F^{\mu\nu J} - \\ &\quad -\frac{1}{2}AB^{-1/2}a_{IJ}\partial_\mu A^I_5\partial^\mu A^J_5 - \frac{1}{\sqrt{6}}C_{IJK}\tilde{F}^I_{\mu\nu}F^{\mu\nu J}A^K_5\,. \end{aligned} \quad \text{(B.9)}$$

Here, we only consider the arbitrary function $A = A(x)$ and $B = B(x)$. To put the equation (B.9) back into the canonical form, we perform a Weyl rescaling of the metric which is given by:

$$g_{\mu\nu} \to A^{-1}B^{-1/2}g_{\mu\nu}\,. \quad \text{(B.10)}$$

So under this rescaling the lagrangian becomes:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\mathcal{L}^{S^1}_{4d} &= \frac{1}{2}R - \hat{a}_{IJ}(\hat{h^I})\partial_\mu\hat{h}^I\partial^\mu\hat{h}^J - \frac{2}{3}\hat{a}_{IJ}(\hat{h^I})\partial_\mu A^I_5\partial^\mu A^J_5 - \\ &\quad -\frac{1}{4}B^{1/2}a_{IJ}F^I_{\mu\nu}F^{\mu\nu J} + \frac{1}{\sqrt{6}}C_{IJK}\tilde{F}^I_{\mu\nu}F^{\mu\nu J}A^K_5\,, \end{aligned} \quad \text{(B.11)}$$

where $\hat{a}_{IJ} = \frac{3}{4}B^{-1}a_{IJ}$ and $\hat{h}^I = B^{1/2}h^I$. The Chern-Simons term vanishes identically for the non abelian case.

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C. Some detailed calculations of the fermionic sectors

The five dimensional spinor Ψ and its conjugate $\overline{\Psi}$ can be written into four dimensional form which are $\Psi \equiv (\psi^1_L, \overline{\psi^2}_R)^T$ and $\overline{\Psi} \equiv (\psi^{2L}, \overline{\psi^1}_R)$. Plugging this definition into (3.49) we get

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[& -\frac{1}{2\kappa_5^2}\psi^2_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^1_{\hat{\nu}} - \frac{1}{2\kappa_5^2}\overline{\psi}^1_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\overline{\psi}^2_{\hat{\nu}} - \\ & - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\psi^2_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^1_{\hat{\nu}} - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\overline{\psi}^1_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\overline{\psi}^2_{\hat{\nu}}\Big]. \end{aligned} \tag{C.1}$$

By using $\psi^1_{\hat{\mu}} = \frac{1}{\sqrt{2}}(\psi^+_{\hat{\mu}} + \psi^-_{\hat{\mu}})$ and $\psi^2_{\hat{\mu}} = \frac{1}{\sqrt{2}}(\psi^+_{\hat{\mu}} - \psi^-_{\hat{\mu}})$, (C.1) can be rewritten as

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[& -\frac{1}{2\kappa_5^2}\frac{1}{2}\Big(\psi^+_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^+_{\hat{\nu}} - \bar{\psi}^-_{\hat{\rho}}\gamma^{\hat{\rho}\hat{\mu}\hat{\nu}}D_{\hat{\mu}}\psi^-_{\hat{\nu}}\Big) - \\ & - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}\Big(\psi^+_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^+_{\hat{\nu}} - \psi^-_{\hat{\mu}}\gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\psi^-_{\hat{\nu}}\Big)\Big] + \text{h.c.} \end{aligned} \tag{C.2}$$

Now we split the space-time coordinates into: $x^{\hat{\mu}} = (x^\mu, x^5)$,

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Bigg[& -\frac{1}{2\kappa_5^2}\frac{1}{2}\Big(\psi^+_\rho\gamma^{\rho\mu\nu}D_\mu\psi^+_\nu + \psi^+_\rho\gamma^{\rho\mu5}D_\mu\psi^+_5 + \psi^+_\rho\gamma^{\rho5\nu}D_5\psi^+_\nu + \\ & + \psi^+_\rho\gamma^{\rho55}D_5\psi^+_5 + \psi^+_5\gamma^{5\mu\nu}D_\mu\psi^+_\nu + \psi^+_5\gamma^{55\nu}D_5\psi^+_\nu + \\ & + \psi^+_5\gamma^{5\mu5}D_\mu\psi^+_5 + \psi^+_5\gamma^{555}D_5\psi^+_5\Big) - \\ & - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}\Big(\psi^+_\mu\gamma^{\mu\nu\hat{\rho}\hat{\sigma}}\psi^+_\nu + \psi^+_\mu\gamma^{\mu5\hat{\rho}\hat{\sigma}}\psi^+_5 + \psi^+_5\gamma^{5\nu\hat{\rho}\hat{\sigma}}\psi^+_\nu + \\ & + \psi^+_5\gamma^{55\hat{\rho}\hat{\sigma}}\psi^+_5\Big) - (+\to -)\Bigg] + \text{h.c.} \end{aligned} \tag{C.3}$$

By inserting the ansatz for the gravitino $\psi^-_\mu = sgn(z)\psi^+_\mu$ and $\psi^-_5 = sgn(z)\psi^+_5$, (C.3) becomes

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[& -\frac{1}{2\kappa_5^2}\frac{1}{2}(1 - sgn(z)^2)(\psi^+_\rho\gamma^{\rho\mu\nu}D_\mu\psi^+_\nu + \psi^+_\rho\gamma^{\rho5\nu}D_5\psi^+_\nu) + \\ & + \frac{1}{2\kappa_5^2}sgn(z)\delta(z)\psi^+_\rho\gamma^{\rho\nu}\psi^+_\nu - \\ & - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}(1 - sgn(z)^2)\psi^+_\mu\gamma^{\mu\nu\hat{\rho}\hat{\sigma}}\psi^+_\nu\Big] + \text{h.c.}\,, \end{aligned} \tag{C.4}$$

where we have used $\partial_5 sgn(z) = 2\delta(z) - \delta(z - \pi R)$. For any values of the $sgn(z)$ we get

$$\begin{aligned} S_{\psi\psi} = \int d^5x\sqrt{-\hat{g}}\Big[& -\frac{1}{2\kappa_5^2}\frac{1}{2}(\psi^+_\rho\gamma^{\rho\mu\nu}D_\mu\psi^+_\nu + \psi^+_\rho\gamma^{\rho5\nu}D_5\psi^+_\nu) - \frac{i\sqrt{6}}{16\kappa_5}h_I\hat{F}^I_{\hat{\rho}\hat{\sigma}}\frac{1}{2}\psi^+_\mu\gamma^{\mu\nu\hat{\rho}\hat{\sigma}}\psi^+_\nu - \\ & - \frac{1}{2\kappa_5^2}\psi^+_\rho\gamma^{\rho\nu}\psi^+_\nu[\delta(z) - \delta(z - \pi R)]\Big] + \text{h.c.} \end{aligned} \tag{C.5}$$

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Next, compactification of the bulk gravitino can be done by inserting the ansatz $\psi_\mu^+ = \frac{1}{\sqrt{2}}C(x)\psi_\mu$ into (C.5) and by using the covariant derivative of the gravitino (2.11), one obtains

$$\begin{aligned} S_{\psi\psi} = & -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\psi_\rho\gamma^{\rho\mu\nu}\left(\partial_\mu\psi_\nu + \frac{1}{2}\omega_\mu{}^{ab}\gamma_{ab}\psi_\nu\right) + \\ & +\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\psi_\rho\gamma^{\rho\mu\nu}\left(-\frac{f_\eta}{f}\partial_\mu\rho + \left(\frac{1}{2\rho}+\frac{f_\rho}{f}\right)\partial_\eta\right)\psi_\nu - \\ & -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\psi_\rho\gamma^{\rho\mu\nu}\left(\partial_\mu \ln C - \frac{1}{4}\partial_\mu \ln B\right) - \\ & -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\frac{i\sqrt{6}}{4\kappa_5}h_I\partial_\rho A_5^I\psi_\mu\gamma^{\mu\nu\rho}\psi_\nu + \text{h.c.} \end{aligned} \tag{C.6}$$

From the bosonic sectors we have obtained the equation (3.36) - (3.40). Inserting these equations into (C.6), one obtains

$$\begin{aligned} S_{\psi\psi} = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}\frac{C^2}{4}\psi_\rho\gamma^{\rho\mu\nu}\Bigg[& \partial_\mu\psi_\nu + \frac{1}{2}\omega_\mu{}^{ab}\gamma_{ab}\psi_\nu + \partial_\mu \ln C\psi_\nu - \frac{1}{4}\partial_\mu \ln B\psi_\nu - \\ & -\frac{i}{2}\Bigg(\left(\frac{2f_S}{f}+\frac{1}{S+\bar{S}}\right)\partial_\mu S - \\ & -\left(\frac{2f_{\bar{S}}}{f}+\frac{1}{S+\bar{S}}\right)\partial_\mu\bar{S}\Bigg)\psi_\nu + \\ & +\left(\frac{1}{4(T^I+T^{\bar{I}})}\partial_\mu T^I - \frac{1}{4(T^I+T^{\bar{I}})}\partial_\mu T^{\bar{I}}\right)\psi_\nu\Bigg] + \text{h.c.}\,, \end{aligned} \tag{C.7}$$

where the Kähler potential (3.45) we have

$$K_S \equiv \frac{\partial K}{\partial S} = -\frac{1}{\kappa_4^2}\left(\frac{2f_S}{f}+\frac{1}{S+\bar{S}}\right), \tag{C.8}$$

$$K_{\bar{S}} \equiv \frac{\partial K}{\partial \bar{S}} = -\frac{1}{\kappa_4^2}\left(\frac{2f_{\bar{S}}}{f}+\frac{1}{S+\bar{S}}\right), \tag{C.9}$$

$$K_I \equiv \frac{\partial K}{\partial T^I} = -\frac{3}{\kappa_4^2}\left(\frac{1}{T^I+T^{\bar{I}}}\right), \tag{C.10}$$

$$K_{\bar{I}} \equiv \frac{\partial K}{\partial T^{\bar{I}}} = -\frac{3}{\kappa_4^2}\left(\frac{1}{T^I+T^{\bar{I}}}\right), \tag{C.11}$$

so that we get the equation (3.55). The spinors in four dimensions are then defined by

$$\psi_\mu = \frac{1}{2}\begin{pmatrix}\psi_{\mu L}\\ \\ \overline{\psi}_{\mu R}\end{pmatrix}, \qquad \overline{\psi}_\mu = \frac{1}{2}\left(\psi_{\mu L}, \overline{\psi}_{\mu R}\right), \tag{C.12}$$

and one obtains

$$S_{\psi\psi} = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-\hat{g}}C^2\bar{\psi}_\rho\gamma^{\rho\mu\nu}\tilde{D}_\mu\psi_\nu\,. \tag{C.13}$$

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Integrating out the above equation with respect to $x^5 \in [-\pi R, \pi R]$, it finally results

$$
\begin{aligned}
S_{\psi\psi} &= -\frac{1}{2\kappa_5^2} \int d^4x \int_0^{2\pi R} dz \sqrt{-\hat{g}} C^2 \bar{\psi}_\rho \gamma^{\rho\mu\nu} \tilde{D}_\mu \psi_\nu \\
&= -\frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \bar{\psi}_\rho \gamma^{\rho\mu\nu} \tilde{D}_\mu \psi_\nu \,, \qquad \text{(C.14)}
\end{aligned}
$$

where we have set $C = A^{-1} B^{-1/4}$, while in the bosonic sector we have taken $A = B^{-1/2}$ such that the covariant derivative of the gravitino in four dimensions is given by (3.58).

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